Angular momentum effects in Michelson - Morley type experiments

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Abstract

The effect of the angular momentum density of a gravitational source on the times of flight of light rays in an interferometer is analyzed. The calculation is made imagining that the interferometer is at the equator of an axisymmetric steadily rotating gravity source. In order to evaluate the size of the effect in the case of the Earth a weak field approximation for the metric elements is introduced. For laboratory scales and non-geodesic paths the correction due to the angular momentum turns out to be comparable with the sensitivity expected in gravitational waves interferometric detectors, whereas it drops under the threshold of detectability when using free (geodesic) light rays.

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1 Introduction

The famous Michelson-Morley experiment does not require any explanation regarding its nature and the crucial role that history reserved to it is well known. It has been analyzed in any respect in the early days of relativity and discussed also on its fundamental meaning [1]. Since then it has been assumed that no anisotropy can be revealed until the frontier of special relativity is crossed. Only in a few cases anisotropies deriving from general relativistic corrections were considered [2][3], but only caused by the gravitational red shift in non-horizontal arms of the interferometer; Schwarzschild-like corrections do not produce any effect in the horizontal plane.

However, if the source of the field is rotating as it is the case for the Earth, the situation in principle changes. This means that a tiny anisotropy can legitimately be expected, depending on the angular momentum of the source.

On the other hand the search for measurable effects of the angular momentum of the gravitational field is always active in order to add a new direct verification of the consequences of general relativity. The only positive result at the moment concerns the precession of the nodes of the orbit of the LAGEOS satellite [4] (Lense-Thirring effect [5]). In the next few years the space mission Gravity Probe B (GPB) is planned to fly carrying gyroscopes which should in turn verify the Lense-Thirring effect too [6]; finally a series of different possibilities connected both with the Sagnac effect and the gravitomagnetic clock effect have been considered [8][7].

The present paper will fix the general formalism to verify a possible influence of the angular momentum density of the Earth on a Michelson-Morley type experiment. Numerical estimates will show that the effect is quite small in any case, however using non-geodesic light paths it could turn out to be comparable with what people are expecting and planning to measure with big size interferometric gravitational waves detectors like LIGO [9] and VIRGO [10].

2 Preliminaries

The Michelson-Morley experiment is an interferometric measure and uses light, let us then start from a generic null line element in polar coordinates and within an axially symmetric static field originated by a central body

endowed with an angular velocity $\Omega = d\phi/dt$:

$$0 = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2 \tag{1}$$

The g's are of course the elements of the metric and are independent both from time t and from ϕ. In weak-field approximation the explicit form of the metric can be [11]

$$\begin{aligned} g_{tt} &\simeq c^2\left(1 - \frac{\mu}{r}\right) \\ g_{t\phi} &\simeq \frac{\mu c}{r} a \sin^2\theta \\ g_{rr} &\simeq -1 - \frac{\mu}{r} + \frac{\sin^2\theta}{r^2} a^2 \\ g_{\theta\theta} &= -r^2 - a^2\cos^2\theta \\ g_{\phi\phi} &\simeq -\left(r^2 + a^2\right)\sin^2\theta \end{aligned} \tag{2}$$

where we introduced the parameters $a = J/Mc$ (J is the angular momentum of the source, M is its mass and c is the speed of light), $\mu = 2GM/c^2$ (Schwarzschild radius of the source). Now let us consider $r =$ constant world lines only. This choice corresponds to limiting the study to light beams contained locally in a "horizontal" plane (actually this would require a wave guide locally shaped as a constant gravitational potential surface). The null (non-geodesic) world line becomes:

$$0 = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2 \tag{3}$$

If we suppose to place our interferometer at the equator ($\theta = \pi/2$) and provided its arms are not too long, the metric (2) (first order in a/r, μ/r and $\frac{\pi}{2} - \theta$) becomes

$$\begin{aligned} g_{tt} &\simeq c^2\left(1 - \frac{\mu}{r}\right) \\ g_{t\phi} &\simeq \frac{\mu c}{r} a \\ g_{rr} &\simeq -1 - \frac{\mu}{r} + \frac{a^2}{r^2} \\ g_{\theta\theta} &\simeq -r^2 \\ g_{\phi\phi} &\simeq -r^2 - a^2 \end{aligned} \tag{4}$$

Consequently for short enough excursions in the "horizontal plane" we can assume, at the lowest order in θ, that, for light, ϕ and θ variations are approximately proportional to each other, so:

$$|d\theta| = \chi\,|d\phi| \tag{5}$$

where χ is a constant.

Suppose now that the interferometer arms are stretched one in the North-South direction and the other in the East-West direction. Taking into account the fact that the Earth reference frame, where the interferometer is at rest, is indeed rotating, the coefficient χ will depend on the angular speed of the Earth.

Now solving (3) for dt we obtain

$$dt = \frac{-g_{t\phi} \pm \sqrt{g_{t\phi}^2 - g_{tt}g_{\theta\theta}\chi^2 - g_{tt}g_{\phi\phi}}}{g_{tt}}d\phi \tag{6}$$

Supposing our light beam starts from a point on the equator at $\phi = 0$ and moves northward, it will be

$$\theta = \frac{\pi}{2} - \chi\phi \tag{7}$$

Of course in the case of an East-West beam it is $\chi = 0$.

3 Times of flight of non-geodesic light beams

In proximity of the equator the first factor in the right hand side of (6) does not depend on θ; thus

$$t_N = \frac{-g_{t\phi} + \sqrt{g_{t\phi}^2 - g_{tt}g_{\theta\theta}\chi^2 - g_{tt}g_{\phi\phi}}}{g_{tt}}\phi_1 \tag{8}$$

t_N is the time of flight to reach the northern mirror and ϕ_1 is the angular coordinate of the event; the drift of the beam is naturally in the prograde sense.

The world line of the mirror (initially at $\theta = \frac{\pi}{2} - \Phi$ and $\phi = 0$; here Φ represents the angular stretch of the interferometer arm) is:

$$t_N = \phi_1/\Omega \tag{9}$$

Ω is of course the angular speed of the Earth.

(8) and (9) allow to deduce an expression for χ:

$$\chi = \frac{1}{\Omega}\sqrt{-\frac{g_{tt} + 2g_{t\phi}\Omega + g_{\phi\phi}\Omega^2}{g_{\theta\theta}}} \tag{10}$$

Actually at the lowest order in θ (10) does not contain θ itself and consequently on an $r = R =$ constant surface χ is a constant.

From (7) we see that to span the South-North angular distance Φ, one travels eastward by the angle

$$\phi_1 = \frac{\Phi}{\chi} \tag{11}$$

Returning to (8) and using (10):

$$t_N = \sqrt{-\frac{g_{\theta\theta}}{g_{tt} + 2g_{t\phi}\Omega + g_{\phi\phi}\Omega^2}}\Phi$$

Considering the North-South way back to the source we see from (8) that it is

$$t_S = \frac{-g_{t\phi} + \sqrt{g_{t\phi}^2 - g_{tt}g_{\theta\theta}\chi_S^2 - g_{tt}g_{\phi\phi}}}{g_{tt}}\phi_2 \tag{12}$$

where ϕ_2 is the Earth's rotation angle between the reflection and the arrival back at the source. It must be

$$t_S = \phi_2/\Omega \tag{13}$$

(12) and (13) give $\chi_S = \chi$ as in (10); then it is $t_S = t_N$, $\phi_2 = \phi_1$. Finally the total time of flight South-North-South is

$$t_{SNS} = t_N + t_S = 2\sqrt{-\frac{g_{\theta\theta}}{g_{tt} + 2g_{t\phi}\Omega + g_{\phi\phi}\Omega^2}}\Phi$$

To proceed further we recall the explicit expressions for the g's, given in (4): now, posing $\Phi = l/R$ where l is the length of the arm of the interferometer and R is the radius of the Earth, one has approximately:

$$t_{SNS} \simeq 2\frac{l}{c}\left(1+\frac{\mu}{2R}+\frac{R^2\Omega^2}{2c^2}+\frac{1}{2}\frac{\Omega^2 a^2}{c^2}\right)-2\frac{\mu a\Omega}{c^2 R}l$$

All further corrections in θ, i.e. Φ, are indeed quadratic and multiply the other small terms, thus resulting much smaller than them.

The next step is to consider the time of flight along the East-West arm of the interferometer.

From formula (3) with $\theta = \pi/2 = \text{const}$ we have

$$t_E = \frac{-g_{t\phi}+\sqrt{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}}{g_{tt}}\phi \tag{14}$$

The world line of the eastern end mirror, assuming equal length arms, is $\phi = \Phi + \Omega t$ which means also

$$\phi_E = \Phi + \Omega t_E \tag{15}$$

Combining (14) and (15) one has

$$t_E = -\frac{g_{t\phi}-\sqrt{\left(g_{t\phi}^2 - g_{tt}g_{\phi\phi}\right)}}{g_{tt}+g_{t\phi}\Omega-\sqrt{\left(g_{t\phi}^2 - g_{tt}g_{\phi\phi}\right)}\Omega}\Phi \tag{16}$$

Now for the way back we have

$$t_W = \frac{g_{t\phi}+\sqrt{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}}{g_{tt}}\left(\phi_E - \phi_W\right) \tag{17}$$

ϕ_W is the angular coordinate of the source at the arrival time of the reflected beam. It must also be

$$\phi_W = \Omega\left(t_E + t_W\right) \tag{18}$$

(18) and (17) together allow for the calculation of t_W:

$$t_W = \frac{g_{t\phi}+\sqrt{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}}{g_{tt}+g_{t\phi}\Omega+\sqrt{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}\Omega}\Phi \tag{19}$$

The total West-East-West time of flight is

$$t_{WEW} = t_E + t_W = 2\frac{\sqrt{g_{t\phi}^2 - g_{tt}g_{\phi\phi}}}{g_{tt} + 2g_{t\phi}\Omega + \Omega^2 g_{\phi\phi}}\Phi \tag{20}$$

The difference in the time of flight along the two arms at the lowest order weak field approximation is

$$\Delta t = t_{WEW} - t_{SNS} \simeq \frac{a^2}{R^2}\frac{l}{c} \tag{21}$$

4 Geodesic light beams

The situation for free, i.e. geodesic, light rays is different from the description given in the previous section. Now we start from the remark that in the equatorial plane the bending of the light rays is lower than the curvature of the circle along which the mirrors of the interferometer move. Actually in the zeroth order of approximation the time of flight of light between the two end mirrors of an arm of the interferometer is deduced from the length of the chord subtended to the appropriate arc of the mirrors circumference. It is

$$t_o = 2\frac{R}{c}\sin\frac{\phi_o}{2} \tag{22}$$

where $\phi_o = \Phi \pm \Omega t$ ($+$ in the prograde path, $-$ in the reverse trip).

Taking into account the effect of the mass M and the angular momentum density a, we expect a deviation from the straight line, which can be expressed in terms of the space curvature of the light beam $k = 1/\rho$ (ρ is the radius of curvature).

A further approximation can be to use the average curvature of the path between the two ends of the interferometer arc. The length of the intercepted beam would then be

$$\rho\psi \tag{23}$$

where ψ is the angle subtending the moving interferometer arc, as seen from the curvature center. When the space trajectory of the light rays is not contained in the equatorial plane, we expect it also no more to be plane at

all, however reasonably the non-planarity corrections will be smaller than the other corrections we are introducing.

The chord subtended to the arc (23) is of course the same when seen from the origin of the reference frame; in the equatorial plane this gives

$$\rho \sin \frac{\psi}{2} = R \sin \frac{\phi}{2}$$

Now ϕ is slightly different from the former ϕ_o:

$$\phi = \phi_o \pm \Omega \delta t$$

where δt is the correction to the time of flight induced by the curvature of the trajectory.

Extracting ψ from (4):

$$\psi = 2 \arcsin \left(\frac{R}{\rho} \sin \frac{\phi}{2} \right)$$

and the time of flight becomes

$$t = \frac{\rho}{c} \psi = 2 \frac{\rho}{c} \arcsin \left(\frac{R}{\rho} \sin \frac{\phi}{2} \right)$$

More explicitly in terms of δt and using (22):

$$\delta t = t - t_o = 2 \frac{\rho}{c} \arcsin \left(\frac{R}{\rho} \sin \frac{\phi_o \pm \Omega \delta t}{2} \right) - 2 \frac{R}{c} \sin \frac{\phi_o}{2}$$

Reasonably it is $\rho >> R$ and all angles are small ($< 10^{-6}$ rad, which is the angle subtended under a 1 m arm, from the center of the Earth). This allows for series developments up to the lowest meaningful orders, thus

$$\delta t = \pm \frac{R}{c} \Omega \delta t + \frac{1}{24} \frac{R^3}{c \rho^2} \left(\phi_o^3 \pm 3 \phi_o^2 \Omega \delta t \right)$$

Solving for δt

$$\delta t = \frac{1}{24} \frac{l^3}{c \rho^2} \tag{24}$$

Now we need an explicit expression for ρ. A standard approach [11] moves from considering the right hand side of (3) divided by $d\lambda^2$ (where λ is

an affine parameter) as the Lagrangian of the light ray. The cyclicity of the t and ϕ coordinates leads to the constants of the motion

$$\begin{aligned} E &= g_{tt}\frac{dt}{d\lambda} + g_{t\phi}\frac{d\phi}{d\lambda} \\ L &= g_{t\phi}\frac{dt}{d\lambda} + g_{\phi\phi}\frac{d\phi}{d\lambda} \end{aligned}$$

wherefrom

$$\begin{aligned} \frac{d\phi}{d\lambda} &= \frac{g_{t\phi}E - g_{tt}L}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}} \\ \frac{dt}{d\lambda} &= \frac{-g_{\phi\phi}E + g_{t\phi}L}{g_{t\phi}^2 - g_{tt}g_{\phi\phi}} \end{aligned}$$

Using the Lagrangian in the equatorial plane one obtains also

$$\left(\frac{dr}{d\lambda}\right)^2 = \frac{2ELg_{t\phi} - E^2 g_{\phi\phi} - L^2 g_{tt}}{g_{rr}\left(g_{tt}g_{\phi\phi} - g_{t\phi}^2\right)} \tag{25}$$

and

$$(r')^2 = \frac{g_{tt}g_{\phi\phi} - g_{t\phi}^2}{g_{rr}} \frac{2jg_{t\phi} - g_{\phi\phi} - j^2 g_{tt}}{\left(jg_{tt} - g_{t\phi}\right)^2} \tag{26}$$

A $'$ denotes differentiation with respect to ϕ and $j = L/E$. Introducing the variable $u = 1/r$ one has

$$\left(u'\right)^2 = u^4 \frac{g_{tt}(u)g_{\phi\phi}(u) - g_{t\phi}^2(u)}{g_{rr}(u)} \frac{2jg_{t\phi}(u) - g_{\phi\phi}(u) - j^2 g_{tt}(u)}{\left(jg_{tt}(u) - g_{t\phi}(u)\right)^2} \tag{27}$$

Now using (2) (27), in the lowest order in the gravitational parameters, becomes

$$\left(u'\right)^2 = \frac{1}{j^2} - u^2 + \mu u^3 + 2\frac{u}{j^3}\mu a + \left(\frac{3}{j^2} - 2u^2\right) u^2 a^2 \tag{28}$$

Differentiating with respect to ϕ we end up with the differential equation

$$u'' + u = \frac{3}{2}\mu u^2 + \frac{1}{j^3}\mu a - 4u^3 a^2 + \frac{3}{j^2} u a^2 \tag{29}$$

Now coming to the local curvature of the rays in the equatorial plane, it is convenient to use a Cartesian reference frame in that plane posing

$$\begin{aligned} y &= \sqrt{-g_{\phi\phi}}\cos\phi \\ x &= \sqrt{-g_{\phi\phi}}\sin\phi \end{aligned}$$

In terms of u (which is a function of ϕ along the trajectory) and using the appropriate approximation:

$$\begin{aligned} y &= \frac{1}{u}\sqrt{(1+a^2u^2)}\cos\phi \\ x &= \frac{1}{u}\sqrt{(1+a^2u^2)}\sin\phi \end{aligned}$$

then

$$w = \frac{dy}{dx} = \frac{u'\cos\phi + (1+u^2a^2)\,u\sin\phi}{u'\sin\phi - (1+u^2a^2)\,u\cos\phi}$$

Finally

$$\frac{1}{\rho} = \frac{dw}{dx} = \mathcal{A}(\phi) + \mathcal{B}(\phi)\,a^2 \tag{30}$$

where $\mathcal{A}(\phi)$ and $\mathcal{B}(\phi)$ are rather complicated functions of u, u', u'' and ϕ.

The formula (30) can be explicitly written in a convenient form if the curvature is calculated at the point of closest approach of the ray to the center of the Earth (maximum value of u which we are calling u_m). There we expect $u' = 0$ and can decide that $\phi = 0$. Reasonably the average value of the curvature along the path of the light differs from the value at the closest approach by small corrections.

The curvature value we shall use is then

$$\frac{1}{\rho} = -\left(u''_m + u_m\right) + \frac{1}{2}\left(u_m + 3u''_m\right)u^2a^2$$

Actually the last term is 12 orders of magnitude smaller than the first, so in practice

$$\frac{1}{\rho} = -\left(u''_m + u_m\right) \tag{31}$$

Now u_m can be obtained equating (28) to 0. At the lowest approximation order it is

$$u_m = \frac{1}{j}$$

Introducing this result into (29) and (31) produces

$$\begin{aligned} \frac{1}{\rho^2} &= \left(\frac{3}{2}\mu\frac{1}{j^2} + \frac{1}{j^3}\mu a - \frac{1}{j^3}a^2\right)^2 \\ &= \frac{9}{4}\frac{\mu^2}{j^4} - 3\frac{\mu}{j^5}a^2 \end{aligned}$$

Now back to the time difference (24). A further crude approximation can be to put $j \simeq R$:

$$\delta t \simeq \frac{1}{24}\frac{l^3}{c}\frac{\mu}{R^4}\left(\frac{9}{4}\mu - 3\frac{a^2}{R}\right) \tag{32}$$

The correction, as we see, is negligibly small: $\sim 10^{-36}$ s for the pure mass term and $\sim 10^{-40}$ s for the a^2 term.

Out of the equatorial plane we cannot reasonably expect the situation to be different. Of course the difference in flight times along the two arms cannot but be less or at most equal in the order of magnitude to (32). In practice this means that for free light rays in the terrestrial environment the straight line approximation is fairly adequate and the time of flight difference has the typical 0 value of the Michelson-Morley experiment.

5 Discussion and conclusion

The result (21) is obtained under the assumption that a physical apparatus (bidimensional wave guide) obliges the light rays to move along constant radius paths. Were this possible the order of magnitude estimate at the surface of the Earth for 1 m long interferometer arms would be:

$$\Delta t \sim 10^{-20}\ s \tag{33}$$

This effect is purely rotational and rather small but not entirely negligible. Should the interferometer rotate in the horizontal plane, the time of flight difference Δt would alternatively change of sign displaying an oscillating behavior which in principle could be detected. In general (33) fixes the scale for a^2 effects on the Earth.

A way to increase the value of (33) would be to have multiple reflections of the light beams happen along each arm of the interferometer before the actual interference is measured. This is what happens in Fabry-Perot type interferometers. Here both arms should be equipped with such devices, it

would then be easy to have the light rays to be reflected back and forth for instance 10^3 or 10^4 times, before the measurement. Since the effect we are looking at is indeed cumulative, this fact would bring the time difference between the two paths to $10^{-17} - 10^{-16}$ s, which corresponds, for visible light, to $10^{-2} - 10^{-1}$ fringe in the interference pattern. This would indeed be a huge effect: $10^{-4} - 10^{-2}$ fractional change of the signal intensity.

It is remarkable that the obtained numeric value compares with the expected phase (and time) shifts in the gravitational wave interferometric detectors now under construction, as LIGO and VIRGO [9],[10]. There indeed a sensitivity is expected, in measuring displacements, of the order of 10^{-16} m which corresponds to a time of flight difference 4 orders of magnitude lower than (33) and consequent much higher sensitivity.

Of course our effect as such is a static one, producing a DC signal and it would be practically impossible to recognize its presence in the given static interference pattern. On the other hand the spectacular sensitivity of gravitational wave interferometric detectors is obtained at frequencies in the range $10^2 - 10^3$ Hz. To extract the information from the background and to profit of highly refined interferometric techniques we would have to modulate the signal. This result could be achieved, for instance, steadily rotating the whole interferometer in the horizontal plane. This solution would however introduce a new source of noise too, in form of vibrations. It would be better to think of a static interferometer with a couple of rotating beams. In practice one could have two cylindric, coaxial mirrors; the internal one should be partially transparent. On the axis one would have a compact rotating head with a couple of source/receivers shooting two light beams orthogonally to each other; this result would actually be performed with an appropriate beam splitter on the axis and a primary source sending light along the axis. The couple of cylindrical reflecting surfaces would act as a Fabry-Perot device; the return beams would interfere on the axis.

Of course in this configuration one would have the sought for signal, modulated at a frequency double of the rotation frequency of the beams. Any imperfection of the mirrors and of the whole set up would of course generate perturbations at the same fundamental frequency as the one of the signal. The difference between signal and rotation induced noise would be that the signal would be peaked on the East-West direction, whereas the various disturbances would have random orientations of their axes. Repeating many measurement runs with different angular configurations of the interferometer and carefully analyzing the data should allow separating the East-West

peaked signal from the rest.

Besides the rotation induced perturbations one would expect also elastic vibration and thermal noise in the solid body of the interferometer. The elastic vibrations can be controlled carefully designing the structure in order to have proper frequencies not coinciding with the rotation frequency of the beams. For a rigid configuration at the scale of the meter, proper frequencies are easily greater than 1 kH.

As for thermal noise, it can be controlled operating at low temperature. In any case if a phase difference can be achieved in the order of one hundredth of a fringe or better, the signal would easily be bigger than the amplitude of the thermal noise of an even moderately cooled device.

In order to cope with the needs both of mechanical and thermal stability, and considering that we need also to properly guide the optical beams a best suited material could be sapphire, whose properties make it extremely interesting for interferometry in gravitational waves detection [12].

The one described above is a simple scheme, showing the principle feasibility of an experiment. A careful analysis of the technical details would of course be needed in order to proceed further.

We can conclude that the calculations we have written in this paper fix the order of magnitude of effects depending from the a^2 of the Earth and show that they should be big enough to be measurable by interferometric techniques.

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